WAKE STIFFNESS AND ITS APPLICATION: TETHERED CYLINDERS AND FLYING SNAKES

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Summary Wake stiffness is the tendency of the wake of an upstream body to pull a downstream body toward the centerline of the wake. Here, we investigate and discuss the validity of applying the wake stiffness model to the dynamics of a tethered pair of cylinders in a uniform flow. We find that wake stiffness provides an effective simplified model for tethered cylinders, but does not explain the whole behavior, particularly for cylinder spacings above 3.5 diameters. We also explore the extension of the wake stiffness concept to modeling the underlying mechanics of flying snakes.

WAKE STIFFNESS IN THE MOTION OF TANDEM, TETHERED CYLINDERS

When considering cylinder oscillations in the wake of an upstream cylinder submerged in a uniform flow, several researchers have found that oscillations persist and even amplify beyond the lock-in region for single cylinders in a flow [2][4]. Assi et al. [1] have proposed an explanation for the effect of the upstream wake that they refer to as "wake stiffness". They demonstrate in their experimental system that the wake of the upstream cylinder acts as a linear spring, pulling the downstream cylinder toward the centerline of the wake.

To understand the behavior of tandem bluff bodies and investigate this concept of wake stiffness, we focused our experimental efforts on investigating tandem circular cylinders. Using a free surface water channel at Virginia Tech, we conducted experiments over a range of cylinder spacings, $x_0/D \in [3.0, 5.0]$, and a range of Reynolds numbers, $Re \in (4\times10^3, 1.1\times10^4)$. In our experiments, the upstream cylinder is fixed and the trailing cylinder is tethered to this fixed cylinder; that is, the downstream cylinder is constrained to move along a circular path around the upstream cylinder, as pictured in Figure 1(a) and shown schematically in Figure 1(c). The trailing cylinder is displaced to angular amplitudes up to $22.1°$, giving a nearly transverse cylinder motion. Since the majority of the literature considers transverse oscillations [1][2][4], we will compare with such experiments.

In their investigations of wake induced vibration of tandem cylinders, Assi et al. [1] conducted experiments on a fixed upstream cylinder with a transversely constrained downstream cylinder on air bearings. They found that steady oscillations of the trailing cylinder persisted when they removed the structural restoring force from their experiments, and they introduce

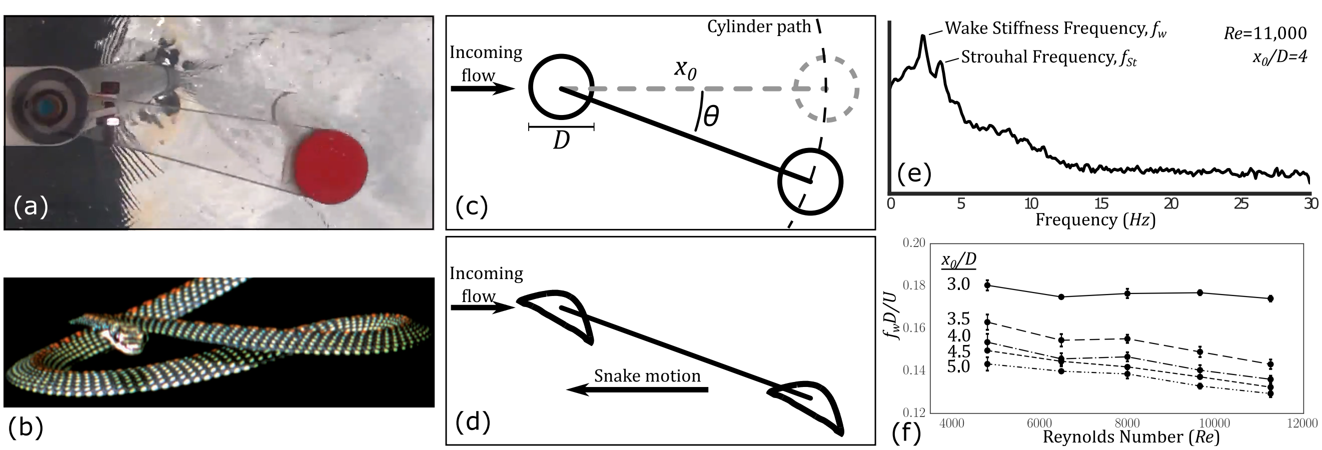

Figure 1: (a) Photo of experimental setup, including grey upstream PVC cylinder and red-capped downstream acrylic cylinder (b) Front view of *C. paradisi* in flight. From Socha [6]. (c) Configuration of the tandem, tethered cylinder system. The trailing cylinder is constrained to move around the upstream cylinder along the "cylinder path." (d) Schematic of side view of tandem wing model of flying snakes. The wings move together and are constrained to a constant distance, giving the similarity to tethered cylinders. (e) Power spectral density (PSD) of dynamic response for cylinder spacing $x_0/D = 4$, showing frequency peaks corresponding to Strouhal frequency and wake stiffness frequency. (f) Apparent Wake Stiffness, f_wD/U, for tandem, tethered cylinders at various spacings. Wake stiffness effects decrease for $x_0/D \geq 3.5$, but remain constant for $x_0/D = 3.0$.

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wake stiffness to explain the frequency of oscillations that they measure. As the cylinder is displaced from the wake centerline, the average magnitude of the coefficient of lift increases, giving the overall wake a stiffness-like behavior for quasi-steady dynamics. The wake stiffness is generated by the mean pressure difference between the surrounding flow and passing vortices shed from the upstream cylinder, providing an approximately linear dependence on displacement.

Since our experiments have no structural restoring force, any forces on the cylinders are due solely to fluid forces. For cylinder separations of $x_0/D \in [3.5, 5]$, the power spectrum of cylinder motion is dominated by two frequencies: a lower power, higher frequency component consistent with the corresponding Strouhal frequency for a stationary cylinder at that flow velocity; and a higher power, lower frequency component that we associate with the wake stiffness, as shown in Figure 1(e). When $x_0/D = 3$, the power spectrum shows one frequency peak. Based on results from the literature [7], this behavior lies within the little-studied gap-flow switching regime for tandem cylinders, which occurs for spacings $x_0/D < 3.5$ according to [7]. As the trailing cylinder is displaced, flow around the cylinders is redirected through the gap, causing the cylinder to be forced back toward the centerline. Assi et al. [1] did not conduct experiments in the gap-flow switching regime.

In the present study, we non-dimensionalize the measured frequency of oscillation by flow velocity and diameter to calculate an apparent wake stiffness, which we define as the stiffness necessary to give the measured frequency. Our experimental results support the use of the wake stiffness model in our tethered cylinder system. Figure 1(f) shows that apparent wake stiffness decreases slightly as Reynolds number increases for all cases within the wake induced vibration regime for which Assi et al. [1] initially postulated wake stiffness. This decreasing relative frequency is also shown in the original paper. In contrast, in the gap-flow switching regime the apparent wake stiffness remains essentially constant throughout the tested range, suggesting that the wake stiffness model is even more applicable to this regime than to the parameter range proposed in [1].

APPLYING WAKE STIFFNESS TO A MODEL OF FLYING SNAKES

A tandem wing model of flying snakes provides an opportunity to extend wake stiffness to tethered, non-circular bluff bodies. Five species of arboreal snakes native to Southeast Asia, most notably *Chrysopelea paradisi*, are able to flatten their bodies into a symmetric airfoil-like shape and glide through the air, as pictured in Figure 1(b). They continue to undulate their bodies throughout the flight much as snakes do on the ground, leading to complex flight mechanics. The flow structure around their bodies is unknown, but Holden et al. [3] have investigated the fluid forces on a representative cross-sectional body shape and have found that the cross-section is a bluff body with a vortex wake at every angle of attack.

In the investigation of flying snakes, work has been done to represent the essential components of the mechanics of gliding with simplified models. One such model, developed by Jafari et al. [5], uses the mean lift and drag forces on the snake body shape from experiments by Holden et al. [3] to influence a tandem wing model, considering the wings as rigidly connected, aerodynamically decoupled bodies, as shown in Figure 1(d). Each wing represents an approximately straight segment of the snake that is transverse to the incoming flow, which is a bluff body at any angle of attack. The tethered cylinder experiments provide a connection between previous cylinder experiments and the rigidly connected snake model. Therefore, we assume that we can apply the concept of wake stiffness to the model, and thereby achieve a first order improvement over the initial model. We find that wake stiffness affects glide stability and distance for the tandem wing model of the snake.

CONCLUSIONS

We examined the validity of the concept of wake stiffness, first introduced by Assi et al. [1], on a tethered cylinder configuration, fixing the distance between the cylinders rather than the direction of oscillation. We found that wake stiffness is a suitable framework for predicting these oscillations, but does not hold completely constant for increasing Reynolds number. The cylinder experiments lay a foundation for exploring the implications of extending the wake stiffness model to a non-circular pair of bluff bodies, such as the cross-sectional body shape of flying snakes.

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